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# On a new observable for measuring the Lense–Thirring effect with Satellite Laser Ranging

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## Abstract

For a pair of twin Earth orbiting artificial satellites placed in identical orbits with supplementary inclinations, in addition to the sum of the residuals of the nodal rates, already proposed for the LAGEOS–LARES mission, also the difference of the residuals of the perigee rates could be employed, in principle, for measuring the general relativistic Lense–Thirring effect. Indeed, on one hand, the gravitomagnetic secular precessions of the perigees of two supplementary satellites in identical orbits are equal and opposite, and, on the other, the classical secular precessions induced by the multipolar expansion of the terrestrial gravitational field are equal, so that their aliasing effect cancels out in the difference of the perigees' rates. If the eccentricities of the two satellites would be chosen to be equal, contrary to the LAGEOS–LARES project, such cancellation would occur at a very accurate level. Among the time-dependent perturbations, the proposed observable would allow to cancel out the even and odd zonal gravitational tidal perturbations and some non-gravitational perturbations. With a proper choice of the inclination of the two satellites, the periods of all the uncanceled time-dependent perturbations could be made short enough to allow to fit and remove them from the signal over observational time spans of a few years. The linear perturbation induced by the terrestrial Yarkovski–Rubincam effect would affect the proposed measurement at a level well below  $10^{-3}$ .

# 1 Introduction

In its weak-field and slow-motion approximation General Relativity predicts that, among other things, the orbit of a test particle freely falling in the gravitational field of a central rotating body is affected by the so called gravitomagnetic dragging of the inertial frames or Lense-Thirring effect. More precisely, the longitude of the ascending node  $\Omega$  and the argument of the perigee  $\omega$  of the orbit [1] undergo tiny secular precessions [2] (The original papers by Lense and Thirring can be found in english translation in [3])

$$\dot{\Omega}_{\text{LT}} = \frac{2GJ}{c^2 a^3 (1 - e^2)^{\frac{3}{2}}}, \quad (1)$$

$$\dot{\omega}_{\text{LT}} = -\frac{6GJ \cos i}{c^2 a^3 (1 - e^2)^{\frac{3}{2}}}, \quad (2)$$

in which  $G$  is the Newtonian gravitational constant,  $J$  is the proper angular momentum of the central body,  $c$  is the speed of light *in vacuum*,  $a$ ,  $e$  and  $i$  are the semimajor axis, the eccentricity and the inclination, respectively, of the orbit of the test particle.

The first experimental check of this predicted effect in the gravitational field of the Earth has been obtained by analyzing a suitable combination of the laser-ranged data to the existing passive geodetic satellites LAGEOS and LAGEOS II [4]. The claimed total relative accuracy of the measurement of the solve-for parameter  $\mu_{\text{LT}}$ , introduced in order to account for this general relativistic effect, is of the order of<sup>1</sup>  $2 \times 10^{-1}$  [4].

In order to achieve a few percent accuracy, in [6] it was proposed to launch a passive geodetic laser-ranged satellite- the former LAGEOS III, subsequently become the LARES [7] - with the same orbital parameters of LAGEOS apart from its inclination which should be supplementary to that of LAGEOS.

This orbital configuration would be able to cancel out exactly the mismodelled part of the classical nodal precessions induced by the multipolar expansion of the terrestrial gravitational field, which are proportional to  $\cos i$  and depend on even powers of  $\sin i$ , provided that the observable to be adopted is the sum of the residuals of the nodal precessions of LARES and LAGEOS

$$\delta\dot{\Omega}^{\text{LAGEOS}} + \delta\dot{\Omega}^{\text{LARES}} = 62\mu_{\text{LT}}. \quad (3)$$

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<sup>1</sup>However, some scientists propose a different error budget [5].

Currently, the observable of the LAGEOS–LARES mission is under revision in order to improve the obtainable accuracy [8].

The orbital parameters of LAGEOS, LAGEOS II and LARES are in Tab. 1.

Table 1: Orbital parameters of LAGEOS, LAGEOS II and LARES.

Orbital parameter	LAGEOS	LAGEOS II	LARES
$a$ (km)	12,270	12,163	12,270
$e$	0.0045	0.014	0.04
$i$ (deg)	110	52.65	70

In this paper we show that the configuration of twin satellites placed in identical orbits with supplementary inclinations can reveal itself more fruitful than that one could have imagined before. Indeed, the sum of the nodes can be supplemented with a new, independent observable given by the difference of the perigees [9]. Of course, such observable would be more difficult to be implemented because, contrary to the nodes, the perigee is a very sensitive orbital element which is affected by many gravitational and non–gravitational perturbations which should be very carefully modelled and treated in the orbital processors like GEODYN II or UTOPIA. However, the great experience obtained in dealing with the perigee of LAGEOS II in the LAGEOS–LAGEOS II Lense–Thirring experiment could be fully exploited in such measurement as well.

The paper is organized as follows. In section 2 we describe such new observable, the impact of the mismodelled static part of the gravitational field of the Earth on it and some possibilities for its practical implementation. In section 3 and 4 we sketch the impact of the non–gravitational and gravitational orbital perturbations on the proposed measurement (a more quantitative analysis with numerical tests can be found in [10]). Section 5 is devoted to the conclusions.

## 2 A new perigee–only observable

The concept of a couple of satellites placed in identical orbits with supplementary inclinations could be fruitfully exploited in the following new way.

An inspection of eq. (2) and of the explicit expressions of the rates of the classical perigee

precessions induced by the even zonal harmonics of the geopotential [11] suggests to adopt as observable the difference of the residuals of the perigee precessions of the two satellites

$$\delta\dot{\omega}^i - \delta\dot{\omega}^{\pi-i} = X_{\text{LT}}\mu_{\text{LT}}, \quad (4)$$

so to obtain a secular trend with a slope of  $X_{\text{LT}}$  mas yr<sup>-1</sup>. Indeed, on one hand, the Lense–Thirring perigee precessions depend on  $\cos i$ , contrary to the nodal rates which are independent of the inclination, so that, by considering the relativistic effect as an unmodelled force entirely adsorbed in the residuals, in eq. (4) they sum up. On the other, it turns out that the classical even zonal perigee precessions depend on even powers of  $\sin i$  and on  $\cos^2 i$ , so that they cancel out exactly in eq. (4). It may be interesting to notice that the proposed observable of eq. (4) is insensitive to the other general relativistic feature which affects the pericenter of a test body, i.e. the gravitoelectric Einstein precession. Indeed, as it is well known [2], it does not depend on the inclination of the satellite’s orbital plane.

In regard to a practical application of such idea, we note that the LAGEOS–LARES mission would be unsuitable because the perigee of LAGEOS is not a good observable due to the notable smallness of the eccentricity of its orbit. For the sake of concreteness, we could think about a LARES II which should be the supplementary companion of LAGEOS II. In this case we would have a gravitomagnetic trend with a slope of -115.2 mas yr<sup>-1</sup> (which is almost twice that of the LAGEOS–LARES node-only mission). Moreover, since the magnitude of the eccentricity of LAGEOS II is satisfactory in order to perform relativistic measurements with its perigee, the LARES II, contrary to the LAGEOS–LARES mission, could be inserted in an orbit with the same eccentricity of that of LAGEOS II. So, the cancellation of the classical secular precessions would occur at a higher level than in the LAGEOS–LARES node-only observable [8]. Of course, a careful analysis of the time-dependent gravitational [13] and, especially, non-gravitational perturbations (see [14] for the radiative perturbations and [15] for the thermal, spin-dependent perturbations), to which the perigee is particularly sensitive, contrary to the node, is needed in order to make clear if also for such perturbations some useful cancellations may occur, and to which extent the uncanceled perturbations may affect the proposed measurement. In the following section we will perform a preliminary investigation: a more detailed analysis has been performed in [10].

### 3 The non-gravitational perturbations

#### 3.1 The radiative perturbations

##### 3.1.1 The direct solar radiation pressure

According to [14], the direct solar radiation pressure does not induce any secular trend on the perigee rate: its signature is long-periodic. Its effect on the difference of the perigee rates of two supplementary satellites amounts to<sup>2</sup>

$$\dot{\omega}_{\text{SRP}}^i - \dot{\omega}_{\text{SRP}}^{\pi-i} = \frac{3A_{\odot}}{8nae} \begin{cases} -2 \cos i \cos \epsilon \cos (\Omega + \lambda + \omega) - \\ 2 \left[ (1 - \cos i) \cos \epsilon \right] \cos (\Omega + \lambda - \omega) + \\ \left[ 2(1 + \cos i) \cos \epsilon \right] \cos (\Omega - \lambda + \omega) + \\ -2 \cos i \cos \epsilon \cos (\Omega - \lambda - \omega), \end{cases} \quad (5)$$

In it  $A_{\odot}$  is the acceleration induced by the direct solar radiation pressure. In the case of a supplementary configuration based on LAGEOS II the harmonic  $\cos(\Omega + \lambda - \omega)$  would induce serious troubles for the proposed measurement of the Lense–Thirring effect. Indeed, on one hand its period amounts to 4,244 days, i.e. 11.6 years, on the other, even by assuming a 0.5% mismodelling in  $A_{\odot}$  [14], the mismodelled amplitude of the perigee rate amounts to 609 mas yr<sup>−1</sup>, while the Lense–Thirring effect is, for the LAGEOS II supplementary configuration, 115.2 mas yr<sup>−1</sup>. The mismodelled amplitude of the perigee perturbation would amount to 750.8 mas. Then, over a reasonable time span of a few years it would superimpose to the relativistic signal and its level of uncertainty would vanish any attempts for extracting the gravitomagnetic signature.

The situation ameliorates if we consider a couple of entirely new laser-ranged satellites of LAGEOS-type with frozen perigees. This means that the inclination would amount to 63.4°, so to make the period of perigee extremely long. A possible orbital configuration could be that in Table 2. In it we quote also the Lense–Thirring effects on the node and the perigee. So, the periodicities of the perturbing harmonics would amount to

$$P(\Omega + \lambda - \omega) = P(\Omega + \lambda + \omega) = 729.56 \text{ days}, \quad (6)$$

$$P(\Omega - \lambda - \omega) = P(\Omega - \lambda + \omega) = -243.8 \text{ days}. \quad (7)$$

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<sup>2</sup>In deriving eq. (5) it has been accounted for the fact that for a couple of supplementary satellites the classical rate of the node changes sign because it depends on  $\cos i$ , while the rate of perigee remains unchanged because it depends on  $\cos^2 i$  and on even powers of  $\sin i$  [11].

Table 2: Orbital parameters of the new supplementary satellites.

$a$ (km)	$i$ (deg)	$e$	$P(\Omega)$ (days)	$P(\omega)$ (days)	$\dot{\Omega}_{\text{LT}}$ (mas/y)	$\dot{\omega}_{\text{LT}}$ (mas/y)
12,000	63.4 (116.6)	0.05	$\mp 733.45$	269,078.41	33	$\mp 44.4$

This is very important because, in this case, over an observational time span of a few years the time-dependent perturbations due to the direct solar radiation pressure could be viewed as empirically fitted quantities and could be removed from the signal.

It should be noticed that the practical data reduction of the perigee rates should be performed very carefully in order to account for possible, unpredictable changes in the physical properties of the satellites' surfaces which may occur after some years of their orbital life, as it seems it has happened for LAGEOS II. Such effects may yield a not negligible impact on the response to the direct solar radiation pressure. However, the great experience obtained in dealing with the perigee of LAGEOS II in the LAGEOS–LAGEOS II Lense–Thirring experiment could be fully exploited for the proposed measurement as well.

### 3.1.2 The Earth's albedo

For the Earth's albedo, which induces only long-periodic harmonic perturbations on the perigee rate [14], the same considerations as for the direct solar radiation pressure hold because the periodicities of the harmonic constituents are the same.

## 3.2 The thermal perturbations

### 3.2.1 The Yarkovski–Rubincam effect

According to [15], the terrestrial Yarkovski–Rubincam effect induces on the perigee rate both secular and long-periodic perturbations. Of course, in regard to the measurement of the secular Lense–Thirring trend the linear Rubincam effect is the most insidious one.

The genuine secular part of the terrestrial Yarkovski–Rubincam perturbation on the perigee rate is, according to [15]

$$\dot{\omega}_{\text{Rub sec}} = \frac{A_{\text{Rub}}}{4na} \cos \vartheta \left[ 1 + 2 \cos^2 i + S_z^2 (1 - 6 \cos^2 i) \right], \quad (8)$$

where  $A_{\text{Rub}}$  is the acceleration due to the Rubincam effect,  $\vartheta$  is the thermal lag angle and  $S_z$  is the component of the satellite's spin axis along the  $z$  axis of a geocentric, equatorial inertial frame.

By assuming, for the sake of generality, for the supplementary satellite a thermal lag angle slightly different from that of its twin, so that  $\vartheta^{\pi-i} = \vartheta^i + \delta$  with  $\delta$  small, the Rubincam secular effect on the difference of the perigee rates of a pair of supplementary satellites becomes

$$\begin{aligned} \dot{\omega}_{\text{Rub sec}}^i - \dot{\omega}_{\text{Rub sec}}^{\pi-i} &= \frac{A_{\text{Rub}}}{4na} \left\{ \left[ (1 + 2 \cos^2 i) + (1 - 6 \cos^2 i) (S_z^{\pi-i})^2 \right] \delta \sin \vartheta + \right. \\ &\quad \left. + (1 - 6 \cos^2 i) \cos \vartheta \left[ (S_z^i)^2 - (S_z^{\pi-i})^2 \right] \right\}. \end{aligned} \quad (9)$$

By assuming  $(S_z^i)^2 = (S_z^{\pi-i})^2$  and  $(S_z^{\pi-i})^2 = 1$ , eq. (9) reduces to

$$\dot{\omega}_{\text{Rub sec}}^i - \dot{\omega}_{\text{Rub sec}}^{\pi-i} = \frac{A_{\text{Rub}}}{4na} (2 - 4 \cos^2 i) \delta \sin \vartheta. \quad (10)$$

According to [15], for LAGEOS II  $A_{\text{Rub}} = -6.62 \times 10^{-10} \text{ cm s}^{-2}$ , so that

$$\frac{A_{\text{Rub}}}{4na} = -1.8 \text{ mas yr}^{-1}. \quad (11)$$

By assuming  $\vartheta = 55^\circ$ , as for LAGEOS, and a mismodelling of 20% on  $A_{\text{Rub}}$ , eq. (10), for an orbital supplementary configuration based on LAGEOS II, yields a mismodelled linear trend of  $\delta \times 0.1 \text{ mas yr}^{-1}$ , so that the relative error in the Lense–Thirring measurement would amount to  $\delta \times (1.4 \times 10^{-3})$ .

On the other hand, by assuming  $\delta \sim 0^\circ$  and  $(S_z^i)^2 \neq (S_z^{\pi-i})^2$ , eq. (9) reduces to

$$\dot{\omega}_{\text{Rub sec}}^i - \dot{\omega}_{\text{Rub sec}}^{\pi-i} = \frac{A_{\text{Rub}}}{4na} (1 - 6 \cos^2 i) \cos \vartheta \left[ (S_z^i)^2 - (S_z^{\pi-i})^2 \right]. \quad (12)$$

With the same assumption as before for LAGEOS II, eq. (12) yields  $\left[ (S_z^{\text{L2}})^2 - (S_z^{\text{LR2}})^2 \right] \times 0.2 \text{ mas yr}^{-1}$  with a relative error in the Lense–Thirring measurement of  $\left[ (S_z^{\text{L2}})^2 - (S_z^{\text{LR2}})^2 \right] \times (1.7 \times 10^{-3})$ . However, it should be noticed that both  $\delta$  and  $(S_z^i)^2 - (S_z^{\pi-i})^2$  could be made very small, so that the presented estimates would become even more favorable. If, e.g. we think about a pair of new, geodetic satellites of LAGEOS type constructed very carefully in the same way and placed in supplementary frozen perigee orbital configuration, it would be quite reasonable to assume their spins as directed mainly along the  $z$  axis, during the first years

of orbital life, as it happened for LAGEOS and LAGEOS II. Numerical simulations confirm such feature. However, it is also important to notice that the difference of the squares of  $S_z$  is involved, so that possible inversions of the rotational motion of the satellites would not yield problems.

According to [15], among the periodicities of the harmonic terms  $2\dot{\omega}$  and  $4\dot{\omega}$  are present. For a couple of new supplementary satellites with frozen perigees the mismodelled part of such harmonics, which, in general do not cancel out in the difference of the perigee rates, would resemble aliasing secular trends. However, their impact on the Lense–Thirring measurement should be at the  $10^{-3}$  level because their amplitude is proportional to  $\frac{(\delta A_{\text{Rub}})}{4na}$  which amounts to  $0.3 \text{ mas yr}^{-1}$ , for LAGEOS II, by assuming a mismodelling of 20% in  $A_{\text{Rub}}$ .

### 3.2.2 The solar Yarkovski–Schach effect

The solar Yarkovski–Schach effect does not induce secular perturbations on the perigee rate [15]. In regard to its long-periodic harmonic terms, which, in general, do not cancel out in the difference of the rates of the perigees of a pair of supplementary satellites, if the frozen perigee configuration would be adopted, their impact was not insidious for the proposed Lense–Thirring measurement. Indeed, as can be inferred from Table 4 of [15], their periodicities do not contain any multiple of the perigee frequency, so that, with the proposed configuration, no semisecular terms would affect the signal.

## 4 The gravitational tidal perturbations

In regard to the orbital perturbations induced by the Earth solid and ocean tides, according to [13], the perigee is particularly sensitive to them, not only to the  $l = 2$  part of the tidal spectrum, but also to the  $l = 3$  constituents.

An important role in assessing the impact of the long periodic tidal perturbations on the proposed measurement of the Lense–Thirring effect is played by their frequencies which are given by

$$\dot{\Gamma}_f + (l - 2p)\dot{\omega} + m\dot{\Omega}. \quad (13)$$

Recall that [13] for the even constituents  $l - 2p = 0$ , while for the odd constituents  $l - 2p \neq 0$ .



Moreover, in eq. (13), for a given tidal constituent of frequency  $f$ ,  $\dot{\Gamma}_f$  depends only on the luni-solar variables. In order to evaluate correctly the impact of the gravitational time-dependent perturbations on the perigee, it is important to note that, according to eq. (36) and eq. (50) of [13], their amplitudes are proportional to

$$\frac{(1 - e^2)}{e} F_{lmp} \frac{dG_{lpq}}{de} - \frac{\cos i}{\sin i} G_{lpq} \frac{dF_{lmp}}{di}, \quad (14)$$

where  $F_{lmp}(i)$  and  $G_{lpq}(e)$  are the inclination functions and eccentricity functions, respectively [12].

Fortunately, the proposed combination  $\dot{\omega}^i - \dot{\omega}^{\pi-i}$  allows to cancel out the 18.6-year and the 9.3-year tides because they are even zonal perturbations. This is an important feature because their extremely long periods are independent of those of the node and/or the perigee of the satellites to be employed: indeed, they depend only on the luni-solar variables. Moreover, their  $l = 2$ ,  $m = 0$  constituents would have large amplitudes, so that, if not canceled out, they would represent very insidious superimposed biasing trends. In regard to the solid and ocean<sup>3</sup>  $l = 2$  tesseral ( $m = 1$ ) and sectorial ( $m = 2$ ) tides, from the fact that their frequencies depend on  $\dot{\Omega}$ , which, as already previously pointed out, changes sign for a supplementary satellite, and from

$$F_{211} = -\frac{3}{2} \sin i \cos i, \quad (15)$$

$$\frac{dF_{211}}{di} = -\frac{3}{2} (\cos^2 i - \sin^2 i), \quad (16)$$

$$F_{221} = \frac{3}{2} \sin^2 i, \quad (17)$$

$$\frac{dF_{221}}{di} = 3 \sin i \cos i, \quad (18)$$

$$(19)$$

it turns out that they do affect  $\dot{\omega}^i - \dot{\omega}^{\pi-i}$ . However, this fact would not have a serious impact on the proposed measurement of the Lense–Thirring effect since the periods of such perturbations would not be too long, so that they could be fitted and removed from the signal over an observational time span of some years.

A careful analysis must be performed for the ocean odd tidal perturbations. Fortunately,

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<sup>3</sup>For the ocean tides we consider only the prograde constituents.

the odd zonal ( $l = 3$ ,  $m = 0$ ) tidal perturbations cancel out. Indeed, for them it turns out that

$$F_{300} = -\frac{5}{16} \sin^3 i, \quad (20)$$

$$\frac{dF_{300}}{di} = -\frac{15}{16} \sin^2 i \cos i, \quad (21)$$

$$F_{301} = \frac{15}{16} \sin^3 i - \frac{3}{4} \sin i, \quad (22)$$

$$\frac{dF_{301}}{di} = \frac{45}{16} \sin^2 i \cos i - \frac{3}{4} \cos i, \quad (23)$$

$$F_{302} = -F_{301}, \quad (24)$$

$$\frac{dF_{302}}{di} = -\frac{dF_{301}}{di}, \quad (25)$$

$$F_{303} = -F_{300}, \quad (26)$$

$$\frac{dF_{303}}{di} = -\frac{dF_{300}}{di}. \quad (27)$$

Moreover, their frequencies are independent of  $\dot{\Omega}$ .

It is important to notice that the same result holds also for the time-dependent perturbation induced on the perigee by the  $J_{2n+1}$  odd zonal harmonics of the geopotential; the  $J_3$  constituent, e.g., depends on odd powers of  $\sin i$ , as it can be noted by the explicit expression of eq. (18) of [16]. This is an important feature because their frequencies are multiple of  $\dot{\omega}$ ; for a frozen perigee configuration they might represent very insidious secular perturbations.

In regard to the  $l = 3$  tesseral ( $m = 1$ ) and sectorial ( $m = 2$ ) tidal lines, it can be proved that they do affect  $\dot{\omega}^i - \dot{\omega}^{\pi-i}$  because their amplitudes depend on inclination functions  $F_{lmp}(i)$  which depend, among other factors, also on  $\cos i$ , contrary to the  $l = 3$ ,  $m = 0$  case. Indeed, it turns out

$$F_{311} = \frac{15}{16} \sin^2 i (1 + 3 \cos i) - \frac{3}{4} (1 + \cos i), \quad (28)$$

$$\frac{dF_{311}}{di} = \frac{15}{8} \sin i \cos i (1 + 3 \cos i) - \frac{45}{16} \sin^3 i + \frac{3}{4} \sin i, \quad (29)$$

$$F_{312} = \frac{15}{16} \sin^2 i (1 - 3 \cos i) - \frac{3}{4} (1 - \cos i), \quad (30)$$

$$\frac{dF_{312}}{di} = \frac{15}{8} \sin i \cos i (1 - 3 \cos i) + \frac{45}{16} \sin^3 i - \frac{3}{4} \sin i, \quad (31)$$

$$F_{321} = \frac{15}{8} \sin i (1 - 2 \cos i - 3 \cos^2 i), \quad (32)$$

$$\frac{dF_{321}}{di} = \frac{15}{8} \cos i (1 - 2 \cos i - 3 \cos^2 i) + \frac{15}{8} \sin i (2 \sin i + 6 \sin i \cos i), \quad (33)$$

$$F_{322} = -\frac{15}{8} \sin i (1 + 2 \cos i - 3 \cos^2 i), \quad (34)$$

$$\frac{dF_{322}}{di} = -\frac{15}{8} \cos i (1 + 2 \cos i - 3 \cos^2 i) - \frac{15}{8} \sin i (-2 \sin i + 6 \sin i \cos i). \quad (35)$$

Moreover, their frequencies are combinations of the form  $\dot{\Gamma}_f \pm \dot{\omega} + \dot{\Omega}$  and  $\dot{\Gamma}_f \pm \dot{\omega} + 2\dot{\Omega}$ . For an orbital configuration based on LAGEOS II this fact would represent a serious drawback because, as pointed out in [13], the  $K_1$ ,  $l = 3$ ,  $m = 1$ ,  $p = 1$ ,  $q = -1$  tidal line induces on the perigee of LAGEOS II a perturbation with nominal amplitude of -1,136 mas and period of 1,851.9 days, i.e. 5.07 years, from the frequency  $\dot{\omega} + \dot{\Omega}$ . The situation is quite similar to that of the direct solar radiation pressure harmonic with a period of 11.6 years. Instead, as in that case, with a frozen perigee configuration the period of the  $K_1$ ,  $l = 3$ ,  $m = 1$ ,  $p = 1$ ,  $q = -1$  tidal line would greatly reduce, so that it could be fitted and removed from the signal over an observational time span of a few years. The same holds also for the other  $l = 3$ ,  $m = 1, 2$  tidal lines.

## 5 Conclusions

In this paper we have proposed to consider the difference of the residuals of the perigee rates  $\delta\dot{\omega}^i - \delta\dot{\omega}^{\pi-i}$ , in addition to the already proposed sum of the residuals of the nodes  $\delta\dot{\Omega}^i - \delta\dot{\Omega}^{\pi-i}$ , as a new observable for measuring the Lense–Thirring effect with a pair of laser–ranged Earth’s satellites of LAGEOS–type in identical orbits with supplementary inclinations. In the well known originally proposed LAGEOS–LARES mission  $\delta\dot{\omega}^i - \delta\dot{\omega}^{\pi-i}$  would be unsuitable because the perigee of LAGEOS cannot be measured accurately due to the smallness of its eccentricity. It should be pointed out that in this paper we have not intended to propose the launch of a couple of new LAGEOS–like satellites in order to measure the difference of the Lense–Thirring perigees’ rates instead of considering the sum of the nodes of the LAGEOS–LARES mission. We have simply investigated, in a preliminary way and from a scientific point of view, if it would have sense to consider the difference of the perigees as a possible, complementary and independent observable for measuring the Lense–Thirring effect with respect to the sum of the nodes: of course, with a couple of new SLR satellites we would have at our disposal both the observables. As it has been shown more quantitatively in [10], the chosen orbital configuration

for the two satellites would yield great benefits also to the measurement of the sum of the Lense–Thirring nodal rates which, of course, would remain much more accurately measured.

A preliminary analysis of the systematic errors induced by the non-gravitational and gravitational orbital perturbations has been carried out. In regard to the gravitational perturbations, also for  $\delta\dot{\omega}^i - \delta\dot{\omega}^{\pi-i}$ , as for  $\delta\dot{\Omega}^i + \delta\dot{\Omega}^{\pi-i}$ , the main systematic error induced by the mismodelled even zonal coefficients of the multipolar expansion of the Earth’s gravitational field cancels out. Also the time-dependent odd zonal harmonics of the geopotential would be cancelled out by the proposed combination. Moreover, the even and odd zonal time-varying tidal perturbations do not affect the proposed observable. This is a very important feature because among them there are the very insidious semisecular 18.6-year and 9.3-year tides, whose frequencies are independent of the satellite’s orbital configuration because they depend only on the luni-solar variables. On the contrary, the tesseral and sectorial tides and most of the non-gravitational time-dependent perturbations do affect  $\delta\dot{\omega}^i - \delta\dot{\omega}^{\pi-i}$ . It is also very important to point out that in the near future the new terrestrial gravity models from CHAMP and GRACE missions will be available, so that the role of the gravitational perturbations in the error budget will notably reduce.

If an orbital configuration based on LAGEOS II and a twin of its, say LARES II, was adopted it would present some drawbacks because of two uncanceled long-periodic harmonic perturbations which have periods of 5.07 years ( $K_1$ ,  $l = 3$ ,  $m = 1$ ,  $p = 1$ ,  $q = -1$  tide) and 11.6 years (direct solar radiation pressure), respectively. Indeed, over observational time spans of a few years they would resemble aliasing superimposed trends which could bias the recovery of the linear Lense–Thirring signal. The optimal choice would be the use of a couple of entirely new geodetic laser-ranged satellites of LAGEOS type accurately constructed in an identical manner with a small area-to-mass ratio, so to minimize the impact of the non-gravitational perturbations, and placed in a frozen perigee configuration. In this way there would not be semi-secular effects and all the time-dependent perturbations affecting the proposed observable would have short enough periods so to be fitted and removed from the signal over reasonable time spans.

The terrestrial Yarkovski–Rubincam effect would induce, among other things, an uncanceled, genuine linear perturbation. This fact is very important because it could mimic the

relativistic trend and make its measurement impossible due to the related level of mismodelling. However, its impact on the proposed measurement of the Lense–Thirring effect would be well below the  $10^{-3}$  level.

However, it should also be considered that the evaluation of the impact of the non-gravitational perturbations on the proposed observable has been worked out on the basis of the currently known status of the physical properties and of the orbital geometries of the existing LAGEOS and LAGEOS II satellites. With a couple of new SLR satellites suitably built up, of course, it would be possible to further reduce the non-gravitational perturbations acting on them for example by reducing their area-to-mass ratio and using rather eccentric orbits. Moreover, it would also be possible to use the data collected just in the first years of their lifetimes so that certain simplifying assumptions on their spin motions could be safely done. The great experience matured with the LAGEOS–LAGEOS II Lense–Thirring experiment in dealing with the time-varying reflectivity properties of LAGEOS II and their impact on the perigee evolution could be fully exploited as well. According to the extensive and conservative numerical analysis of [10], the total impact of the non-gravitational perturbations should amount to almost 5%. Last but not least, the concept of supplementary satellites in identical orbits could be implemented also with a pair of drag-free satellites thanks to recent developments of such technique which could assure a lifetime of many years (M.C.W. Sandford, private communication, 2002).

In conclusion, the proposal of measuring the Lense–Thirring effect with a supplementary pair of satellites turns out to be enforced, at least in principle, because it would be possible to analyze not only  $\delta\dot{\Omega}^i + \delta\dot{\Omega}^{\pi-i}$ , as in the originally proposed LAGEOS–LARES proposal, but also  $\delta\dot{\omega}^i - \delta\dot{\omega}^{\pi-i}$ , provided that a carefully selected orbital configuration is adopted. Moreover, if the new satellites to be launched had rather eccentric orbits, we would have at our disposal both their perigees, apart from that of the existing LAGEOS II, in order to built up suitable combinations of orbital residuals, including also the nodes, which would allow to cancel out many mismodelled even zonal harmonic coefficients, as done in the LAGEOS–LAGEOS II Lense–Thirring experiment. Finally, we would also be able to perform other gravitational tests concerning, e.g., the relativistic gravitoelectric perigee advance [17] and the hypothesis of a fifth force [18].

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